

Index

Indexing is ‘by concept’ as well as by letter: we often include references to passages where a given concept is in play, even if the word or phrase in question does not explicitly appear.

Page numbers accompanied by ‘n’ are references to footnotes.

Our capitalization policy is governed largely by personal taste and is not entirely consistent. Terms capitalized throughout the book typically appear capitalized below; those capitalized in the story but not the commentaries are usually listed here in their uncapitalized form.

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